1999–2000 CATS ASSESSMENT
Open-Response Item Scoring Worksheet

Grade 11 – Mathematics

The **academic expectations** addressed by the open-response item "Storage Tank Measurement" are:

1.5-1.9 Students use mathematical ideas and procedures to communicate, reason, and solve problems.

2.9 Students understand space and dimensionality concepts and use them appropriately and accurately.

2.10 Students understand measurement concepts and use measurement appropiately and accurately.

2.8 Students understand mathematical procedures and use them accurately and appropriately.

The **core content** addressed by this item includes:

MA-H-2.3.4 Geometry and Measurement (Relationships): Students will understand how a change in one or more dimensions of a geometric shape affects perimeter, area, volume, or surface area.

MA-H-2.3.1 Geometry and Measurement (Relationships): Students will solve real-world geometry problems by using algebra.

MA-H-1.3.4 Number and Computation (Relationships): Students will understand how ratio and proportion can be used in a variety of mathematical contexts and to solve real-world problems.

Storage Tank Measurement

The Mattox Oil Company has a 90-foot-tall cylindrical storage tank that contains 210,000 gallons of oil when it is full.

a. Find the radius of the tank. Show your procedure.

($V = \pi r^2 h$ and 1 gallon = 0.1337 cubic feet)

b. How tall would the tank in **part a** have to be in order to double the volume? Show your procedure.

c. How would the volume of the tank in **part a** be affected if both the radius and height of the tank were doubled? Explain your answer.

SCORING GUIDE
Grade 11 Mathematics

Score	Description
4	Student scores 4 points.
3	Student scores 3 - 3.5 points.
2	Student scores 2 - 2.5 points.
1	Student scores .5 - 1.5 points. **OR** Student shows some ability to use the formula for finding the volume.
0	Response is totally incorrect or irrelevant.
Blank	No response.

Note: At the 4 level, student must include correct units for part (a) and part (b).

SCORE POINTS

Part a 1 point Student finds correct radius with work shown.

OR

.5 point Student finds incorrect radius due to calculation error only.

Part b 1 point Student finds correct height with work shown.

OR

.5 point Student finds incorrect height due to calculation error only.

Part c 1 point Student correctly states the volume would be increased by a factor of 2^3 or 8.

OR

.5 point Student correctly states the volume would be increased by a factor of approximately 8.

1 point Student provides correct explanation or correct work for correct answer.

OR

.5 point Student shows incomplete explanation or work.

Note: Student who only states that volume gets bigger gets no points for part c.

SCORING GUIDE
Grade 11 Mathematics

ANSWERS

Part a $210{,}000 \times 0.1337 = 28{,}077$

$28{,}077 = \pi r^2 90$

$r^2 = 99.3$

$r = 9.96$ or $r = 9.97$ feet ≈ 10 ft.

Part b $V = \pi r^2 h$ OR $2 \times 90 = 180$

$(420{,}000 \times 0.1337 = 56{,}154)$

$56{,}154 = \pi(9.965)^2 h$

$56{,}154 \div \pi(9.965)^2 = h$

180 ft. $= h$

Part c Volume would increase by a factor of 2^3 or 8.

Possible explanations:

(1): $V = \pi(2r)^2 \times (2h)$

$V = 8\pi r^2 h$

(2): $V = \pi r^2 h$ with ($r = 2 \times 9.965$ & $h = 2 \times 90$)

$V = \pi(19.93)^2 \times (180)$

$V = 224{,}614.08$ cubic feet

$224{,}614.08 / 28{,}077 = 8$

Student might add:

\# gallons = 224,614.08 cubic feet / 0.1337 gal/cu.ft.

\# gallons = 1,679,985.6

1,679,985.6 gallons / 210,000 gallons = 7.9999 or 8

ANNOTATED STUDENT RESPONSE
Grade 11 Mathematics

Sample Student Response Scored a 4

Student Response

A. h = 90 ft.

V = 210,000 gallons =

$$\left(\frac{210{,}000 \text{ gallons}}{1}\right)\left(\frac{.1337 \text{ cubic feet}}{\text{gallon}}\right) =$$

28077 cubic ft.

$V = \pi r^2 h$

$28077 = \pi \bullet r^2 \bullet 90$

$\frac{28077}{90 \bullet \pi} = r^2$

$\sqrt{\frac{28077}{90 \bullet \pi}} = r = 9.96757\ldots$ ft.

Student determines the correct radius with correct and complete work shown. (1 point)

B. $V = \pi r^2 h$

$\pi r^2(2h) = 2(\pi r^2 h) = 2V$

h would have to be twice as much,

so it would be $90 \times 2 = 180$ ft.

Student determines the correct height with correct and complete work shown. (1 point)

C. $V = \pi r^2 h$

$\pi \bullet (2r)^2 \bullet 2h = \pi 4r^2 \bullet 2h$

$= 8(\pi r^2 h) = 8 \times V$

Volume would be 8 times as large. This is because if the radius is doubled then squared, it is the same as the radius squared times 4. Since that would be 4 times double the volume, (in "B", it was found that double height is also double volume) that would come out to 8 times the volume.

Student correctly states that the volume would be increased by a factor of 8 and clearly explains (algebraically and in words) why. (1 + 1 = 2 points)

Overall, the student earns 4 points, demonstrating a strong understanding of space and dimensionality concepts and measurements and a strong ability to use these concepts and measurements to accurately determine volume in real-world situations.

ANNOTATED STUDENT RESPONSE
Grade 11 Mathematics

Sample Student Response Scored a 4

Student Response

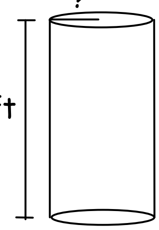
a) $210{,}000(.1337) = \pi r^2(90)$

$28{,}077 \text{ ft.}^3 = 90\pi r^2$

$\frac{311.966 \text{ ft}^2}{\pi} = \frac{\pi r^2}{\pi}$

$99.30 \text{ ft.}^2 = r^2$

$9.97 \text{ ft.} = r$



Student determines the correct radius with correct and complete work shown. (1 point)

b) $2V = \pi r^2 h$

$2(28{,}077) = \pi(9.97)^2 h$

$\frac{56154}{99.3\pi} = \frac{99.3\pi h}{99.3\pi}$

$180 \text{ ft.} = h$

Student determines the correct height with correct and complete work shown. (1 point)

c) $V = \pi(2r)^2 2h$

$V = \pi 4r^2 2h$

$V = 8\pi r^2 h$

The volume would multiply by 8. The "r" is squared, so when you double it, $(2r)^2$ becomes $4r^2$. The $4r^2$ multiplies with the $2h$ to create $8\pi r^2 h$, or 8 times the original value.

Student correctly states that the volume would be increased by a factor of 8 and clearly explains (algebraically and in words) why. (1 + 1 = 2 points)

Overall, the student earns 4 points, demonstrating a strong understanding of space and dimensionality concepts and measurements and a strong ability to use these concepts and measurements to accurately determine volume in real-world situations.

Sample Student Response Scored a 3

Student Response

a) $210000g \times .1337\ ft^3 = 28077\ ft^3$

$$\frac{28077\ ft^3}{90} = \frac{\pi r^2 (90\ ft)}{90}$$

$$\frac{311.97}{\pi} = \frac{\pi r^2}{\pi}$$

$99.302 = r^2$

$\sqrt{99.302} = r$

$r = 9.965\ ft$

Student determines the correct radius with correct and complete work shown. (1 point)

b) $$\frac{56154}{311.964} = \frac{\pi (9.965)^2 h}{311.964}$$

$h = 180\ ft$

Student determines the correct height with correct and complete work shown. (1 point)

c) $v = \pi(20)^2(180)$

$v = 226194.67\ ft^3$

Obviously, the volume would increase as the radius and height were doubled, as is shown when the problem is worked out. By doubling the height, you double the volume. By doubling the height and radius, the volume is approximately 8 times larger than it originally was.

Student states that the volume would increase by a factor of “approximately 8” and attempts to explain (algebraically and in words) why. The explanation, however, is incomplete. (.5 + .5 = 1 point)

Overall, the student earns 3 points, demonstrating a general understanding of space and dimensionality concepts and measurements and a general ability to use these concepts and measurements to accurately determine volume in real-world situations.

ANNOTATED STUDENT RESPONSE
Grade 11 Mathematics

Sample Student Response Scored a 2

Student Response

a. $V = 210{,}000(.1337)$

$V = 28077$

$\frac{28{,}077}{90\pi} = \frac{\pi(r^2)(90)}{90\pi}$

$\frac{311.9\overline{66}}{\pi} = r^2$

$99.30207 = r^2$

$9.96504 = r$

Student determines the correct radius with correct and complete work shown. (1 point)

b. $v = 28077(2)$

$v = 56154$

$\frac{56154}{99.30207\pi} = \frac{(\pi)(99.30207)(h)}{99.30207\pi}$

$565.486701 = h$

Student determines incorrect height due to a calculation error (probably forgot to enter the division by pi into the calculator). (.5 point)

c) $v = \pi(397.208089)(180)$

$v = 224615.8826$

All you would do is double your Radious and substitute it into your equation ($v = \pi r^2 h$). Then you would double your height and substitute it into your equation. After you have done this you will multiply pie by your radious squared and your height. The result will be your new volume.

Student attempts to answer part c but never states that the volume would be increased by a factor of 8. Student offers an explanation (algebraically and in words), but the explanation is incomplete. (.5 point)

Overall, the student earns 2 points, demonstrating some understanding of space and dimensionality concepts and measurements and some ability to use these concepts and measurements to accurately determine volume in real-world situations.

ANNOTATED STUDENT RESPONSE
Grade 11 Mathematics

Sample Student Response Scored a 1

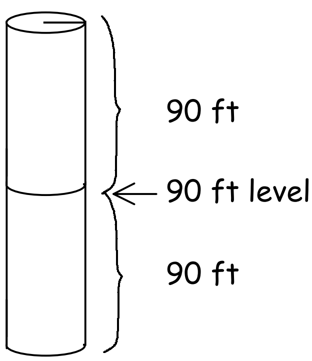
Student Response

A. 210,000 gal × $\frac{.1337 \text{ ft}^3}{1 \text{ gal}}$ =

28077 cubic ft. = $\pi \cdot 9.58^2 \cdot 90$

$r^2 = 9.58$

B.



You would double the height because a tank the same size would hold the same amount of water.

C. It (The volume would be quadrupled because you would be multiplying the cylinder by 2^2. (2 squared)

Student does not determine the correct radius due to a calculation error. (.5 points)

Student determines the correct height, although 180 ft. is not explicitly stated, but rather correctly diagrammed and explained. (1 point)

Student states the incorrect scale factor for the volume (i.e., "quadrupled") and gives an incorrect and incomplete explanation. (0 points)

Overall, the student earns 1.5 points, demonstrating a minimal understanding of space and dimensionality concepts and measurements and a minimal ability to use these concepts and measurements to accurately determine volume in real-world situations.

INSTRUCTIONAL STRATEGIES
Grade 11 Mathematics

The open-response item **"Storage Tank Measurement"** was designed to address students' ability to (1) perform operations with real numbers (including irrational numbers) in problem-solving situations, (2) calculate volume of cylinders in problem settings using formulas, and (3) understand how a change in one or more dimensions of a geometric shape affects volume. The instructional strategies below present ideas for helping students explore and master these concepts and skills.

Provide opportunities for students to work individually, in pairs, in small groups, and/or as a class to complete (with teacher guidance and support) any or all of the following activities:

- Make boxes and/or cylinders. The frame of reference might be designing cereal containers. Students might fill their containers with cereal, popcorn or other "filler" to determine volume. They could vary the height and base to determine maximum volume with a given surface area. Then double or triple the surface area and determine the new volume, comparing it to the old volume.
- Use the hands-on work to connect to an abstract, symbolic situation. Determine the optimal dimensions of a container to yield a fixed volume with a given surface area. Students can work cooperatively to cut and measure their containers. They can discuss the posed questions, modify and clarify their problem-solving steps, and then attach appropriate mathematical notations to their models and solutions. They should explain and justify their steps in writing. The problem may be posed to include data collection and graphing.
- Explore real-world problems that include converting from one unit of measurement to another using conversion factors within a measurement system.
- Explore problems that involve computation with irrational numbers such as square roots and π.